Technical Notes

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Computation of the Potential Flow over Airfoils with Cusped or Thin Trailing Edges

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Introduction

THE method of singularities provides a simple and fast way to compute the two-dimensional potential flowfield around general airfoils. However, the nonuniqueness of the type and distributions of singularities that satisfy the boundary conditions may require a lot of know-how to carry out an accurate computation. According to Hess,[1] the best results are obtained by putting the singularities (sources/sinks and vortex sheet) right on the surface. The numerical accuracy of the result is fairly good for conventional airfoils but deteriorates for airfoils such as the laminar NACA five-digit wing sections, which have a thin and cusped trailing edge. A demonstrative example is shown in Fig. 1 relative to a NACA 65012 at 10-deg incidence. Note that the surface pressure exhibits a nonrealistic loop at the trailing edge, and the overall lift coefficient is 10% lower than the exact value. Increasing the number of elements from 40 to 100 does not improve the result. To cure the problem, Hess[1] recommends replacing the usual constrant-strength vortex distribution by a variable-strength distribution approaching zero at the trailing edge. Numerical experiments have been conducted with a constant gradient of the vortex intensity along the chord. An overall improvement is observed when the gradient increases. However, an asymptotic tendency appears only for very large gradients. The vorticity then changes sign from the leading to the trailing edge. The justification of the procedure is thus questionable. A rational and simple method of dealing with such airfoil shapes is presented in this Note, which may be applied even to airfoil sections with zero trailing-edge angles.

Analysis of the Problem

The link between the airfoil shape and the structure of the matrix of the linear problem to be solved may be useful in pointing out the origin of the problem. The method of solution in terms of velocity is briefly described.[1-3]

The imaginary perturbation velocity induced at the midpoint of the Ith element by the N element of the airfoil is simply

$$V_I^* = (u - jv)_I = \sum_{j=1}^{N} A_{IJ}\sigma_J - j\sum_{j=1}^{N} B_{IJ}\gamma_J \qquad (1)$$

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where ( )* denotes the complex conjugate, A_{IJ} and B_{IJ} are the influence coefficients, and σ_J and γ_J are the source and vortex densities, respectively, of the element J. If the vortex density γ is a constant, Eq. (1) may be simplified to

$$V_I^* = \sum_{J=1}^{N} A_{IJ}\sigma_J - j\gamma B_I \qquad (2)$$

The boundary conditions are the N conditions of zero total normal velocity at the control points and the Kutta condition at the trailing edge:

$$Jm[(1+V_I^*)\cdot \mathrm{d}S_I] = 0, \quad I = 1,N \qquad (3)$$

$$Re[(1+V_1^*)\cdot \mathrm{d}S_1] = Re[(1+V_N^*)\cdot \mathrm{d}S_N] \qquad (4)$$

$\mathrm{d}S$ is the unit surface vector.

If the induced velocities in Eqs. (3) and (4) are replaced by expression (2), a set of $(N+1)$ linear equations are obtained that can be solved to calculate $\sigma_1 ... \sigma_N$ and γ. The typical structure of the matrix is sketched in Fig. 2. It has a strong main diagonal since $|A_{II}| \geq |A_{IJ}|$ for $I \neq J$ but is not usually diagonally-dominant. Large off-diagonal terms can affect the accuracy of the solution if a direct method of inversion is used. This can occur even for classical airfoils, and a simple solution is to replace the conventional clockwise numering of the elements by an alternate numbering (see Ref. 3).

Let us now examine the problems arising from small or cusped trailing edges.

Small Trailing-Edge Angle

In the limiting case of zero trailing-edge angle, the 1th and Nth elements are superposed. Therefore, the boundary condition on one of the two elements is automatically fulfilled by the other. The matrix is singular since two rows are identical. This limiting case arises only for zero trailing-edge; however, it is obvious that the numerical accuracy of the solution will degenerate as the trailing-edge angle becomes small.

Cusped Trailing Edge

The existence of an inflection point on the airfoil surface does not produce a singularity. If several elements are on the same line, however, the normal velocity induced by one of these elements on the others is zero. Therefore, the matrix contains very small terms close to the main diagonal, and the accuracy of the solution can be poor. Notice that the two kinds of problems are closely related (cusped and small trailing-edge angles).

Method of Solution

The proposed solution of the computational problems arising from the shape of the trailing-edge region is to modify the airfoil geometry by means of a simple conformal mapping. The solution for the transformed airfoils is computed numerically. This can be done with the basic code, and the results are then transposed to the physical plane through direct transformation. The two consecutive mappings are not

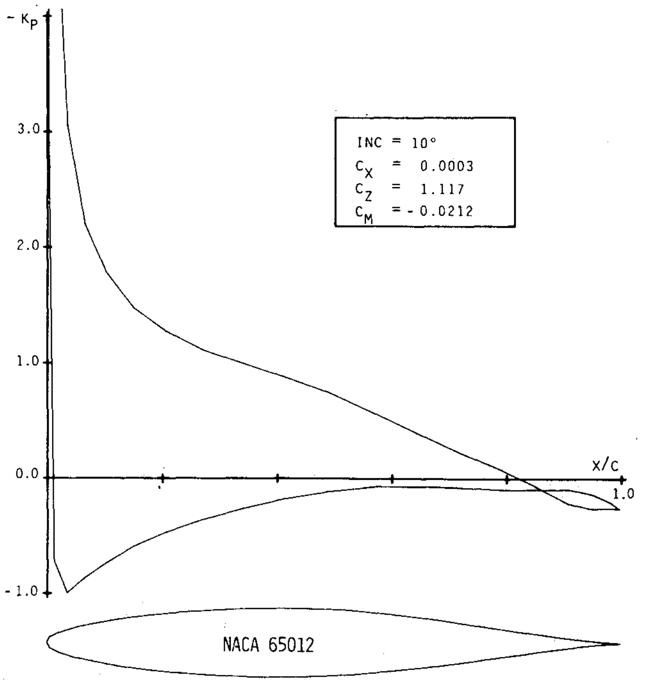

Fig. 1 Pressure distribution over a NACA 65012 airfoil as computed using the conventional method (surface singularities: sources/vortex).

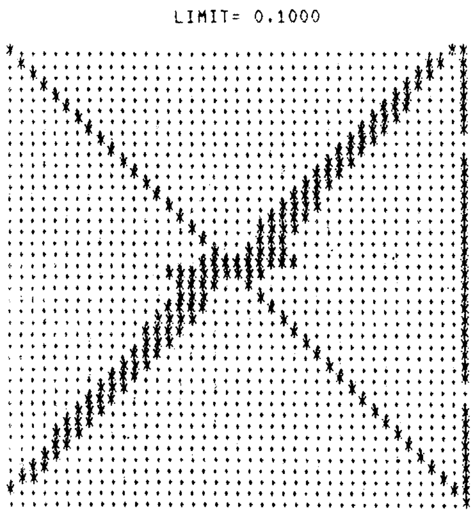

Fig. 2 Structure of the matrix of the linear system to be solved: * and + denote terms larger than and less than 10%, respectively, of the diagonal term (corresponding row).

Table 1 Lift coefficients at 10-deg incidence compared to the analytical result

Wing section	NACA 65012	JK 015
C_ℓ conventional method	1.117	1.032
C_ℓ present method	1.217	1.207
$C_\ell = 2\pi(1+0.77t/c)\sin\alpha^5$	1.192	1.217

time-consuming and may be inserted as optional subroutines to the main program.

The direct Joukowski transformation

$$\xi = z + 1/z \tag{5}$$

transforms a circle of radius $R = 1+\epsilon$ in the z plane into a symmetrical Joukowski airfoil, provided that the center lies on the real axis and that the circle passes through the point

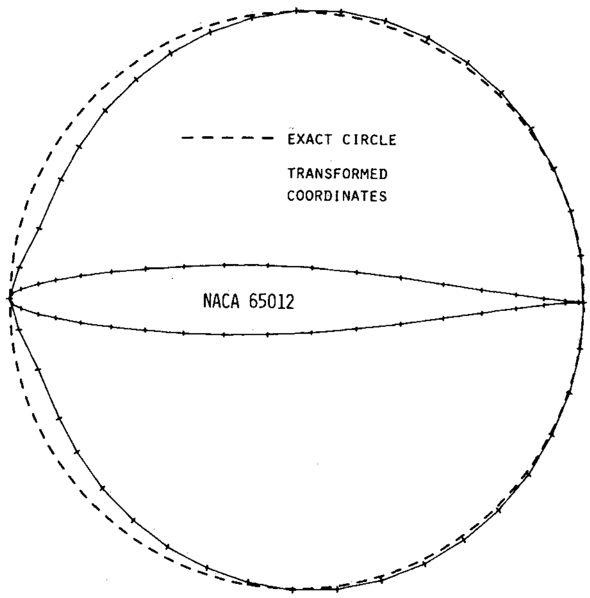

Fig. 3 Geometry of the transformed airfoil (Joukowski transformation).

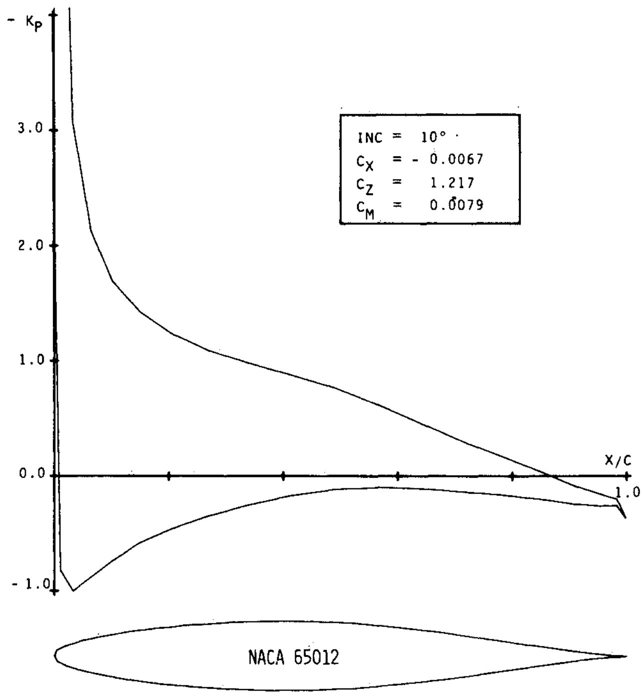

Fig. 4 Pressure distribution over a NACA 65012 airfoil as computed using the method of singularities in the transformed plane.

[1.,0]. For simplicity, this presentation is limited to symmetrical airfoils. Curved airfoils can be handled in a similar fashion; the center of the circle need only be shifted from the real axis.[4] The airfoil obtained has the following properties:

Localization of the TE: [2.,0.] (6)

$$\text{LE: } [(-2-4\epsilon^2+8\epsilon^3+..),0.] \tag{7}$$

$$\text{Thickness at midchord: } \left.\frac{t}{c}\right|_{0.5} = \epsilon - \frac{\epsilon^2}{2} - \epsilon^3 + \frac{\epsilon^4}{2} + \ldots \tag{8}$$

To map a given airfoil into an approximate circle ϵ can be computed by relation (8) truncated to ϵ^2:

$$\epsilon = 1 - \sqrt{1 - 2t/c} \tag{9}$$

The coordinates of the airfoil are stretched such that the trailing edge is located at [2.,0.] and the leading edge at [(−2. −4ϵ^2),0.]. The transformed coordinates are then calculated by means of the inverse relation (5):

$$z = \zeta \pm \sqrt{\xi^2 - 4}/2 \tag{10}$$

The sign of the square root depends on the quadrant.

The transformed airfoil has the shape shown on Fig. 3 for a NACA 65012_1 wing section. The structure of the matrix for the linear system is now quite satisfactory, with a strong main diagonal and no large off-diagonal terms.

Once the solution has been computed in the z plane, the physical solution can be easily obtained by transforming the coordinates and vectors according to relation (5). The local pressure coefficients computed by the present method are presented in Fig. 4. The pressure loop at the trailing edge has completely disappeared, and a large increase in the load coefficient is observed. Typical results for 10-deg incidence are listed in Table 1.[5]

References

[1]Hess, J. L., "Review of Integral-Equations Techniques for Solving Potential-Flow Problems with Emphasis on the Surface-Source Method," *Computer Methods in Applied Mechanics and Engineering*, No. 5, 1975, pp. 145-196.

[2]Basu, B. C. and Hancock, G. J., "The Unsteady Motion of a Two-Dimensional Aerofoil in Incompressible Inviscid Flow," *Journal of Fluid Mechanics*, Vol. 87, Pt. 1, 1978, 159-178.

[3]Bousquet, J., *Methode des Singularités*, Cepadues Editions, Toulouse, France.

[4]Duncan, W. J., Thom, A. S., and Young, A. D., *Mechanics of Fluids*, Edward Arnold Publishers, London, 1970.

[5]Schlichting, H. and Truckenbrodt, E., *Aerodynamik des Flugzeuges*, Springer-Verlag, Berlin, 1967.

Cancellation Zone in Supersonic Lifting Wing Theory

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Introduction

BASING their work on a linear theory, Evvard[1] and Krasilshchikova[2,3] independently developed an expression that yields the perturbation generated by a thin lifting wing of arbitrary planform flying at supersonic speed on a point placed on the wing plane inside its planform,[1] or both on and above the wing plane.[2] This point must be influenced by two leading edges, one supersonic and the other partially subsonic. Although these authors followed different approaches, their methods concur in showing the existence of a perfectly defined cancellation zone.

In this Note, the Evvard approach is generalized to the case solved by Krasilshchikova. Circumventing the latter's lengthy and somewhat complex approach, Evvard's simple method seems to be useful at least for educational purposes.

The Proof

Consider the configuration shown in Fig. 1. In a lifting problem, the perturbation velocity potential at point N, placed on the wing plane but outside its planform and wake, must vanish; that is,

$$\psi(x_N, y_N, 0) = -\frac{1}{\pi} \iint_{S_1+S_2} \frac{w(x_0,y_0)\mathrm{d}x_0\mathrm{d}y_0}{\sqrt{(x_N-x_0)^2 - \beta^2(y_N-y_0)^2}} = 0 \tag{1}$$

where x_0,y_0 represent the source-point coordinates, $\beta = (M_\infty^2 - 1)^{1/2}$, and M_∞ is the freestream Mach number. w is the vertical velocity, which will be denoted w_i or w_o for source points placed inside or outside the planform, respectively. Transformation to the characteristic coordinate system r,s defined by

$$r = x - \beta y, \quad s = x + \beta y, \quad \bar{z} = \beta z, \quad \frac{\partial(x_0,y_0)}{\partial(r_0,s_0)} = \frac{1}{2\beta}$$

$$r_0 = x_0 - \beta y_0, \quad s_0 = x_0 + \beta y_0 \tag{2}$$

leads to

$$\iint_{S_1+S_2} \frac{w(r_0,s_0)\mathrm{d}r_0\mathrm{d}s_0}{\sqrt{r_N - r_0}\sqrt{s_N - s_0}} = \int_0^{r_N} \frac{\mathrm{d}r_0}{\sqrt{r_N - r_0}}$$

$$\left[\int_{s_0=B'O(r_0)}^{s_0=OB(r_0)} \frac{w_i(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N-s_0}} + \int_{s_0=OB(r_0)}^{s_0=s_N} \frac{w_o(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N-s_0}}\right] = 0 \tag{3}$$

The right-hand side of Eq. (3) is an Abel equation equated to zero, so that the terms in brackets should vanish for $r_0 = r_N$; i.e.,

$$\int_{s_0=OB(r_N)}^{s_0=s_N} \frac{w_o(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N-s_0}} = -\int_{s_0=B'O(r_N)}^{s_0=OB(r_N)} \frac{w_i(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N-s_0}} \tag{4}$$

as the integrand is known to have no singularities in the region of integration. This equation is the basis of the determination of the cancellation zone. The direct Krasilshchikova approach requires the inversion of the integral equation (4), followed by a double integration, as in Eq. (1). These steps, however, are not essential for the demonstration and, as shown in Ref. 1 for the case of a point on the wing planform, Eq. (4) can be used to determine the cancellation zone for a point $P(x,y,z)$ placed outside the wing, which is the purpose of this Note.

Let us calculate the perturbation potential at P. The region of integration is divided into three parts, S_0, S_1, S_2, as shown in Fig. 2. $P'(x,y,0)$ is the projection of P over the wing plane, and the curve S is the intersection of the fore cone of P with the wing plane. In characteristic coordinates, the perturbation potential reads

$$\psi(r,s,\bar{z}) = -\frac{1}{2\pi\beta} \iint_{S_0+S_1+S_2} \frac{w(r_0,s_0)\mathrm{d}r_0\mathrm{d}s_0}{\sqrt{(r-r_0)(s-s_0) - \bar{z}^2}} \tag{5}$$

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